“X-problem of number 3”. Definition…

A. Kornyushkin
kornju.hop.ru, kornju@mail.ru

Annotation. Shown “X-problem of number 3” for one dimension and related observations .

1 Introduction

Before theory of numbers dispose an extensive area of "Automata Machine Configuration 2x3" and one interesting problem. Let’s formulate the definition of it for one dimension case.

2 Reversible cellular machine on the graph

Let’s be mixed, finite graph $G = (V, E, A)$. (Author used determination from this resource http://en.wikipedia.org/wiki/Graph_%28mathematics%29, all except determination is not needed there!)

Here $V = \{v_i\}$ is the node of graph, $E = \{e_i\}$ directed edges and $A = \{a_i\}$ undirected. Our graph is ordinary and hence do not contains loops and multiple edges. (Briefly review, that directed edges - is the arrow from one node to another, undirected is simply junction among the nodes. We can tell that there is two arrows leading from here till there and from there till here).

Definition 1.

If there exists closed route on directed edges (or undirected!)

$$v_0 \xrightarrow{a_0 \vee e_0} v_1 \xrightarrow{a_1 \vee e_1} \ldots \xrightarrow{a_k \vee e_k} v_0$$

in which you can get around all of the nodes of the graph and return back, so let's call this graph Super Weak computable.

Definition 2.

If there is a route

$$v_0 \xrightarrow{a_0} v_1 \xrightarrow{a_1} \ldots \xrightarrow{a_k} v_k$$

only by undirected edges – with which you can also get around all the nodes – so let's call this graph Weak computable. So it is clear: if the graph is Weak computable it is also Super Weak computable.

Let the nodes of the graph be in three color (A, B and C).

$$v_i \in \{A, B, C\}$$

So we enter two definitions

Transliteration of graph G $(\tilde{G})$ we can name colors of graph G where all nodes in color C, recolour into color B, and all nodes in color B recolor into C.

$$G(B \Leftrightarrow C) \equiv \tilde{G}$$

Complement of graph G $(\overline{G})$ we can name colors of graph G where all nodes in color A, recolour into color B, and all nodes in color B recolor into A. (Usually this rule is for events when there is no nodes colored into C).

$$G(A \Leftrightarrow B) \equiv \overline{G}$$

Let's enter the rule of transformation of colors in graph G:

$$G_t \rightarrow G_{t+1}$$

It depends on existence directed or undirected edges starting from this node and leading to another node, *at least one node colored into C, or not*? (Let's call this conclusion as P-conclusion).

So if P=0 (there is no color C colored nodes) then recolouring is

$$\begin{matrix} P = 0 \\ (I) \end{matrix} \Rightarrow \begin{matrix} v_t = A \Rightarrow v_{t+1} = A \\ v_t = B \Rightarrow v_{t+1} = C \\ v_t = C \Rightarrow v_{t+1} = B \end{matrix}$$

(let's give a name this recolouring as rule I).

And if P=1 (reverse event) realizing next way of recolouring

$$\begin{matrix} P = 1 \\ (II) \end{matrix} \Rightarrow \begin{matrix} v_t = A \Rightarrow v_{t+1} = C \\ v_t = B \Rightarrow v_{t+1} = A \\ v_t = C \Rightarrow v_{t+1} = B \end{matrix}$$

(let's give a name this recolouring as rule II). It is just some kind of reversible cellular automation from [1]).

Then…

Theorem 1.

$$\tilde{G}_t \rightarrow \tilde{G}_{t-1}$$

Proof.

For proving enough to just consider all five events (shown by numbers in circles) from figure 1.

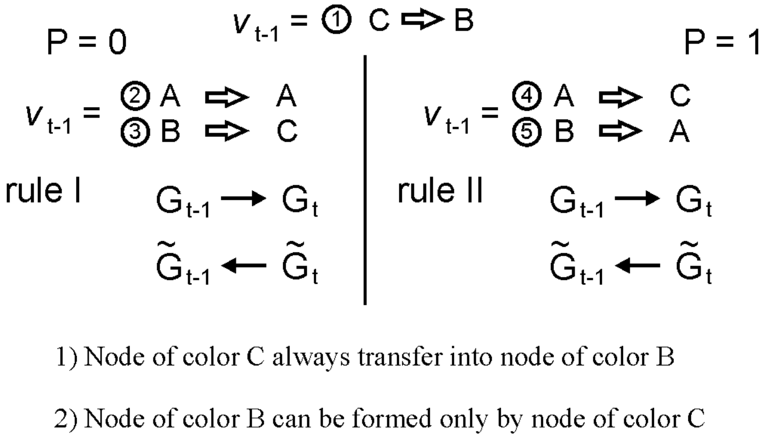

Fig. 1.

Let's take event (1). It is obvious. If we do transliteration, then CB will be BC.

Let's take event (2). Recolouring rule I, then among the node of graph where directed (or undirected) edges leads from node v_{t-1} there is no colored into C. Then in the derivative of graph on its nodes would be no B colored. (Node of color B can be formed only from nodes of C). Then in transliterated graph, among at all its nodes would be no C nodes. That means, transfer rule would be I. And v_t in graph $\tilde{G}_t$ will transfer into A as it was necessary .

Least 3 events are considered the same way. The theorem is a simple consequence of 2 statements shown on figure 1.

Let action of recolouring of graphs will start at t = 0 time. And let starting colouring nodes will consist of only two colors: A and B.

That means that next step (there is no nodes C; the rule on a whole graph is I) and graph will transfer into itself transliteration, consisting only of A and C colors . (All nodes of color B will transfer into C, all nodes of color A will stay constant) . And graph "reverse time" (see Theorem 1); and all the next steps at the t = n* = - n + 1 moment our graph will be equal to transliteration of graph in time n. (See figure 2).

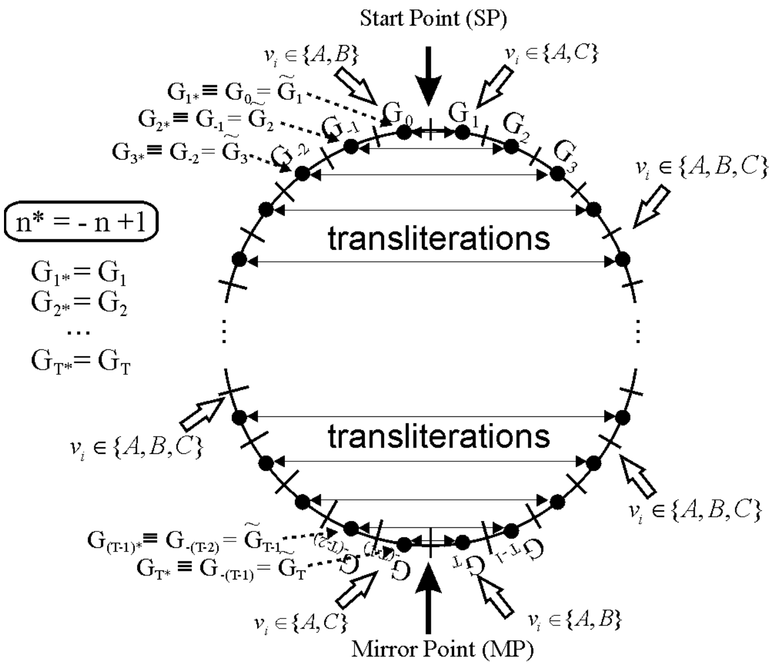

Fig. 2. Motion of recolouring of graph G.

Let's call the point, between the conditions of graph at the time 1* and 1 Start Point (SP)… and continue the motion in two direction: by the time and reverse. Somewhere (because our graph defined as finite)… at time T* - T these two motions will meet at point G_{T*} - G_T, and G_{T*} will be the transliteration G_T. So say, that we measure T – **period** (or back period) and call this point – Mirror Point (MP). (Here we use the main proper of reversible finite Automats. That any finite reversible Automat “obligated by starting point”). That is natural. If in any recoloured graph has exactly one previous (and it is easy to find: let’s make transliteration, then one step forward, then transliteration again), so any recolourings of graph are cyclic).

If T > 2 then Mirror Point exists. And in this case, full reverse period (2T) is an even number. (Otherwise in Mirror Point graph is transliteration of itself. And that means that all nodes of graph are coloured into A, and graph is simply stays).

Let’s prove next theorem.

Theorem 2. (About coloring of Mirror Point).

Let T > 2 and graph is Super Weak computabe. And Start Point – is state between graph colouring in colors A and B (G_{1*}) and graph colouring into A and C (G_1).

Then Mirrow Point is state between graph colouring in colors A and B (G_T) and graph colouring into A and C (G_{T*}).

Proof.

If in graph G_T at the moment of time T would be at least one node coloured into C, then in the basements leading to it edges must be coloured into C (recolouring rule II) too. (That means that colour A transfer into C, also nodes B into A. But none of them are not transliteration of each other). And go around all the nodes along the route into “anoter side” – we can make a conclusion that all graph consist of nodes C. And graph transfers “all nodes is C ” – “all nodes is B ”, “all nodes is B” – “all nodes is C”… (T< 3). Contradiction.

Then the theorem of *extisting of number* λ is true.

Theorem 3.

If our graph is Weak computable, then for any of our nodes v_i, on the motion from SP to MP –

$$v_{i,t=1} \to v_{i,t=2} \to \ldots \to v_{i,t=T}$$

will meet equal number of cases of nodes colored into A (N_A), and B (N_B), and C (N_C). (And the quantity of nodes colored into B and C will be equal).

$$N_B = N_C \equiv N_{BC}$$

Let’s call the value $\lambda = N_A - N_{BC}$.

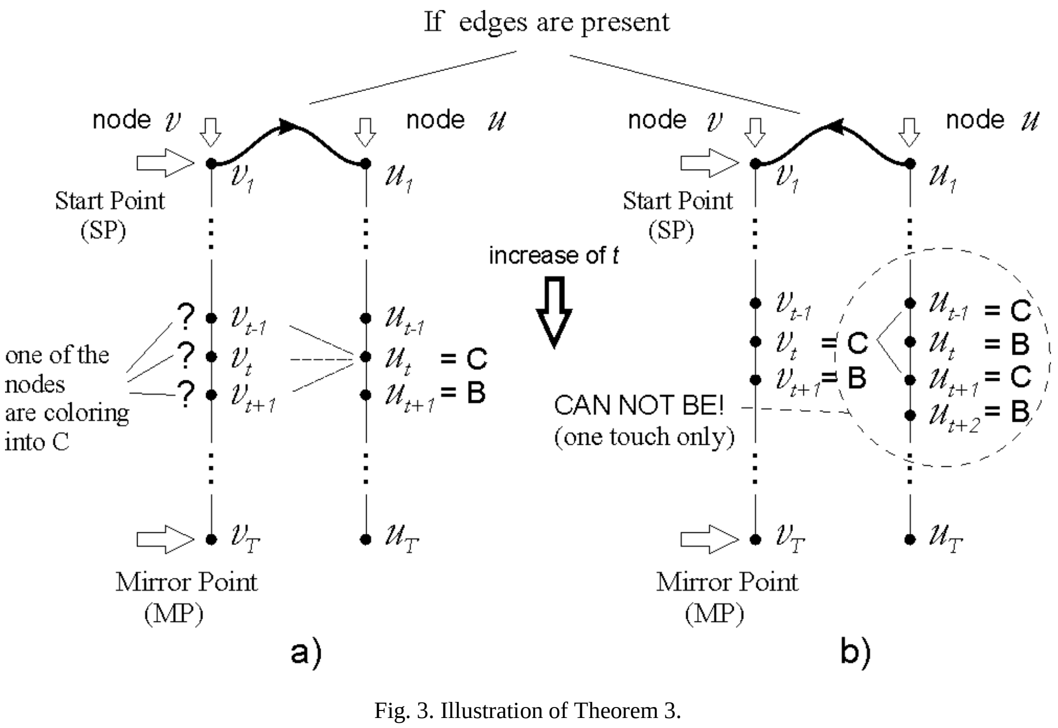

Fig. 3. Illustration of Theorem 3.

Proof.

Let the nodes v and u are bounded "double sided" – undirected edges.

Lets dispose nodes and their current colors on time axis – from Start Statement to Mirror. See figure 3.

Let's pretend chess and chess moves which has no limit on move on any undirected edges and move on time axis one module measure maximum.

Look at the figure 3a. From "directed half" edges leading from node v into node u and if u_t, colored into C, then one of the nodes v_{t-1}, v_t, v_{t+1}, would be colored into C also. (Let's pretend reverse of thesis and ask which color would be node marked on figure 3a by central issue (node v_t)? If it is colored into A, then node under it (node v_{t+1}) colored into C. (Transfer rule – II). Into a color B it also could not be colored. Then node abovt it v_{t-1}, would be colored into C. And into color C node (v_t) also can not be painted! Then it is contradiction).

As our chess figure can step back (get back on start point), that means that it fracture all nodes of graph G from t = 1 to t = T colored into C on set of classes $\bigcup\{C_{v,t}\}$. (Class $\{C_{v,i}\}$ - is nodes multiplied by time set, then our pretending chess figure can pass from one node to another in a few steps).

Our graph is Weak computable and from pic 3a we can say that our chess figure can pass all nodes . That means that any set consists all of nodes.

But from fig. 3b we can make a conclusion that any nodes exists on each $\{C_{v,i}\}$ just one time!

Actually … Nodes colored into C on next step transfer into B. And "double touch" is impossible. From "directed half" leading from node u to node v. u_t = B can't transfer into u_{t+1} = C. Transfer rule II must be. And in this case u_{t+1} must be A. Contradiction again.

From C will allways transfer to B we can make a conclusion that $N_B = N_C$.

And $N_A = T - 2{*}N_{BC}$ (quantity of nodes painted A) also equal for all nodes.

Theorem is proved.

Take Weak computable graph. Consider …

G_{1*}, consisting of color A and B and having it own T and λ…
and complement graph **(!)**
$\overline{G}_{1*}$ (also, naturally, consisting only from A and B colors) and having it own $\overline{T}$ and $\overline{\lambda}$.

Let's take any node v and will consider all array of colors $v(t)$, for a first graph G_{1*}, from time SP- MP. Its size is T.

Let's enter four arrays $a_v(t)$, $b_v(t)$, $c_v(t)$, $\lambda_v(t)$; where t = 0..T–1.

$a_v(t)$= 1 if $v(t)$=A, and $a_v(t)$=0 otherwise. The same way for $b_v(t)$, $c_v(t)$. And

$$\lambda_v(t) = \sum_{0\le p\le t}^{p} (2a_v(p) + b_v(p) + c_v(p) - 1)$$

Let's define three arrays $C_v(k)$, $A_v(k)$, $f_v(k)$. (array "f " call "integral phase").

$C_v(k)=0$; but if we will find t, which will have value $k=\lambda_v(t)/2$ – integer number and that $v(t)=C$, then $C_v(k)=1$.

If $C_v(k)=1$ then we can say that $f_v(k)=t$. Otherwise it "undefined" ($f_v(k)=-1$).

The same way as $C_v(k)$ array, we can find array $A_v(k)$.

$A_v(k)=0$; but if we will find t, which will have measure $k=(\lambda_v(t)-1)/2$ – integer number and that $v(t)=A$, then $A_v(k)=1$.

And we can define these arrays for color $\overline{G}_{1*}$: . $\overline{a}_v(t), \overline{b}_v(t), \overline{c}_v(t), \overline{\lambda}_v(t), \overline{A}_v(t), \overline{C}_v(t), \overline{f}_v(t)$.

Now…

If the value $T+\overline{T}$ dividing into 3 and $K=(T+\overline{T})/3$ …

And further 7 ratio actions are right

$$G_T = \overline{G}_{\overline{T}} \qquad [1]$$

$$\lambda = -\overline{\lambda} \qquad [2]$$

$$\overline{T} - T = \lambda \qquad [3]$$

And for any node *v* and $k = 0..K-1$

$$C_v(k) + \overline{C}_v(k) = 1 \quad [4]$$
$$A_v(k) + \overline{A}_v(k) = 1 \quad [5]$$
$$\overline{A}_v(k) = C_v(k) \quad [6]$$
$$\overline{C}_v(k) = A_v(k) \quad [7]$$

… and additionally!
From ratio [4] we can define full integral phase modulo 2

$$F_v^{(2)}(k) = \begin{cases} f_v(k) \bmod 2 & \text{if } f_v(k) \neq -1 \\ 2 + \overline{f}_v(k) \bmod 2 & \text{otherwise} \end{cases}$$

Then for all nodes v and u additionally needed to make next ratios

$$F_v^{(2)}(0) \equiv 0 \pmod 2$$
$$F_v^{(2)}(2k\text{-}1) \equiv F_v^{(2)}(2k) \pmod 2$$
$$\textit{and if } K \textit{ is even}$$
$$F_v^{(2)}(K\text{-}1) \equiv F_u^{(2)}(K\text{-}1) \pmod 2$$

(we can call these formulas “eighth” ratio).
If all eight ratios are right, then we can make a conclusion that G_{1*} and $\overline{G}_{1*}$ goes from SP to MP with keeping of **Invariant of Precise Filling (IPF)**.

3 “X-problem of number 3” in one dimension. Definition

We can tell that L nodes of graph arranged in a circle and numbered: x = 0..L-1.

Edges of graph defined by two integer numbers n (conventionally “left”) and m (conventionally “right”).

Existing only edges (x,(x-(i+1)*b_i(n)) mod L) and (x,(x+ (i+1)*b_i(m))mod L) for all x and i. (Here b_i(n) – items i of binary expansion of number n; n=b_0(n)+ b_1(n)*2+…+ b_p(n)*2^p). “Module L” showing loop of circle. Give a name to a such a construction **Automat of Configuration 2x3** (AC23) in one dimension.

Let’s suppose that n and m are odd. Then our Automat is Weak computable.

Suppose that there exist such n an m that Automat **at any L** and **any beginning colors** G_{1*}, and $\overline{G}_{1*}$ – goes from SP to MP **with keeping IPF**. Let’s call it **correct mask**. In other case **incorrect**. And from here ….

“X-problem of number 3” in one dimension.

Prove (or disprove?) **that the masks (1.1). (1,3), (3,5), (3,3), (5,5) ... – are correct**. (For example, for the mask (1.5), it is known that it was false). Full list (apparently?) correct masks to the values n, m = 39 is shown in figure 4.

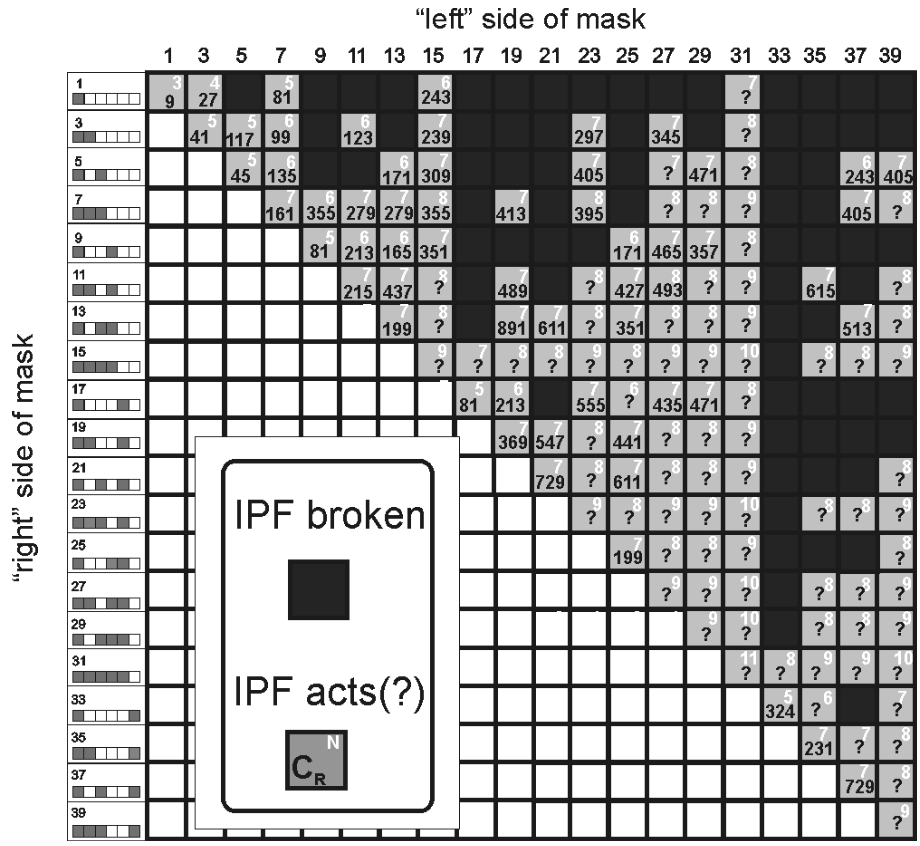

Fig. 4. Table showing which Weak computable masks is correct(?) (shown in gray) and incorrect (shown in black). All centrally symmetric (the cells on the main diagonal of the table) – is correct(?).

We illustrate, first, "the ratio number 8" for the correct masks. See Figure 5.

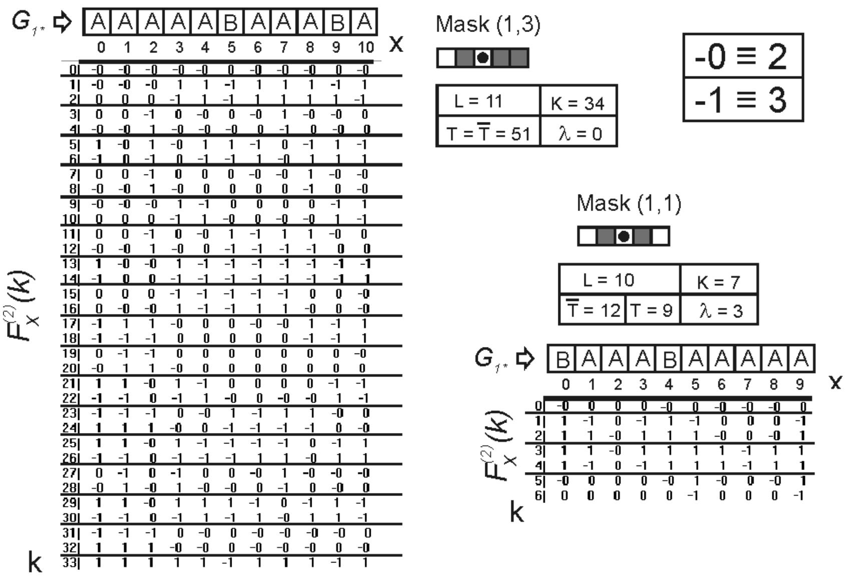

Fig. 5. Illustration of ratio number 8 for correct masks (1,3) and (1,1).

It is evident that after the first row – our filling is in “pairs of lines”. In Figure 5, the last line of the mask (1.3) has an odd value. It is not necessary. They may be even.

Imagine outgoing, very powerful observation, which, perhaps, help us to solve some day "X-problem." For this we consider a complete integrated phase modulo 3. The function $F^{(3)}_x(k)$.

$$F^{(3)}_x(k) = \begin{cases} f_x(k) \bmod 3 & \text{if } f_x(k) \neq -1 \\ 3 + \overline{f}_x(k) \bmod 3 & \text{otherwise} \end{cases}$$

Then it turns out…

that the next line is unambiguously determined from the previous one, facing *the values in the mask above it*. (Thus, for us, the point of the mask include a "central" point). And the number of whole configurations which leads to the values 0, 1 and 2 *are equal*. See Figure 6. We call this algorithm **"resolution"**; the corresponding table for the numbers 0, 1, 2 – **"resolution"** table; and the number of rows in our table (C_p) – **“resolution"** constant for each correct mask.

That is, with the first line and the resolution table we can directly, without difficulty, restore the entire function $F^{(3)}_x(k)$. And therefore matrix C_v and A_v. The criterion that we have reached the end (to MP), is precisely equation $F^{(3)}_x(k) \bmod 3 = 0$ to entire line (for all x).

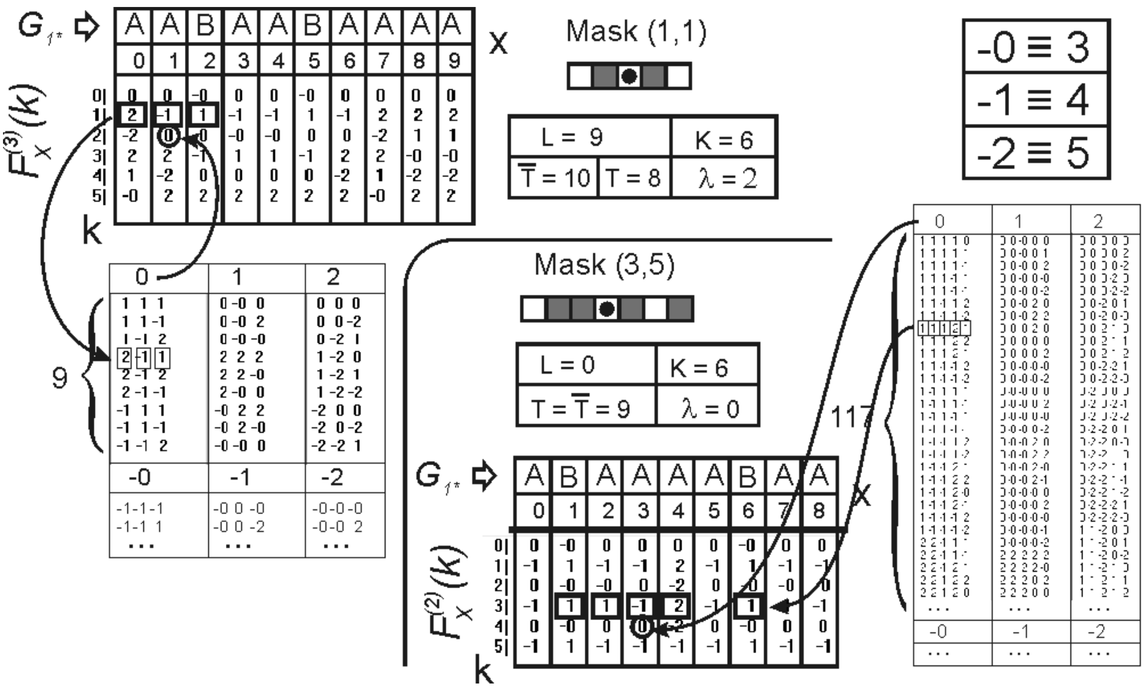

Fig. 6. Illustration of the main observations for masks (1,1) and (3,5). The figure shows the resolution table for the mask (1,1) and the beginning of the table for the mask (3.5) for the numbers 0, 1, 2. Tables for the numbers -0 (3), -1 (4), and -2 (5) can be obtained automatically, as a "complement" of the original.

Moreover, it is clear that the resolution table have the property of "additive". That is, if one mask covers the other, then its resolution table includes a covered table.

This means that there is an *infinite* resolution table in one dimension. There is ***no doubt*** that a similar "X-problem of number 3” is present ***in all dimensions*** [2].

Some interesting aspects of the "X-number of problems 3" in two dimension see in [3].

You can download illustrative program here kornju.hop.ru .

Literature

[1] Tommaso Toffoli and Norman Margolus, *Cellular Automata Machines: A new environment for modeling* , MIT Press 1987), 259 pp.;

[2] О новой математике: Автомате Чистой Тройки, A. Kornyushkin, LAP Lambert Academic Publishing - ISBN: 978-3-659-33017-9

[3] About a Discrete Cellular Soliton (computer simulation) A. Kornyushkin http://arxiv.org/abs/1109.4552