X-problem of number three, definition

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Received 23.08.2013

Abstract

Presented is the notion that a large number of objects (author called them masks) demonstrate a suspiciously large number of identical properties (identities). The connection between the identities is hard-wired and can be implemented through so-called Invariants of Precise Filling. The author verified the existence of these identities and also showed that even in one dimension it involves number 3. Author points out that even more amazing identities related to number 3 can exist in N dimensions, so the problem of proving their existence in N-dimensional space is called by author as "the X-problem of number 3". Number 3 is directly related to reversible cellular Automata with three states. The author used an approach based on reversible cellular Automata and computing for demonstration of existence of such identities in one dimension.

Keywords: discrete mathematic, Numbers theory, cellular automata, graph, reversibility

1. Introduction

Mathematicians and physicists are well aware of a large number of positive attributes associated with the number three.

The author reminds that stable orbits in a central field with a potential inversely proportional to the distance from the object to the center of the force (e.g., gravity field or Coulomb field) exist only in a three-dimensional (3D) space. Yes, the 3D space has a *non-sequiturial* property: any perturbation in a position or velocity of the orbiting object leads to a new stable and well-defined orbit. This phenomenon can be described by corresponding solutions for the motion of a point charge in a Coulomb field in 3D space. There are no similar equations in other realms of mathematics that manifest such an amazing property as those describing the motion in 3D space.

We can ask the question: are there other dimensions (except the 3D one) where the vector product has "good" properties, including the most useful *Jacobi identity***.** The answer is no, only in three dimensions the vector product has good (that is, practical) properties [1]. Therefore, the lack of the useful (from the viewpoint of conservation laws) concept for the rotor in these dimensions, renders the ND (N≠3) vector algebra simply out of practical consideration.

We can provide more examples, for instance, in physics we can see following fundamental combinations: two by three of leptons, two by three of quarks; three colors in Quantum Chromodynamics (QCD), *etc.* Here, the author decided to go further and ask is there a *direct(!)* connection between the *properties of number 3* and *N-dimensional integer lattice.*

The author decided to gain an insight into this generalized problem of "dimension of three" using a computer and obtained a very interesting answer. The most unusual is that the author started his approach from a distance, more particularly he started from *reversible Cellular Automata*.

A lot of remarkable properties of the *Cellular Automata* has been discovered and listed in the book written by Stephen Wolfram [2]. Yet, here we do not need to consider *reversible Automata* of a general form, just as a special case, so-called *cellular Automata* of second-order.

The concept of the second-order Automata was introduced by Edward Fredkin and then studied by other authors [3]-[5].

As shown by Toffoli and Margopolus [6], any second-order automaton may be transformed into a conventional cellular automaton where the transition function depends only on a single previous time step. It handles just three types of cells. We will not use the original definitions as "ready", "excited" and "refracted", but will refer to three types of the cells as A, B and C.

2. Reversible cellular machine on the graph

Let us prove three following simple theorems and give one important definition.

Consider mixed, finite graph $G=(V,E,A)$. We are using the definition from the following resource: http://en.wikipedia.org/wiki/Graph_%28mathematics%29. Below, we will use only this definition and no other developments from the graph theory.

Here $V=\{v_i\}$ is the node of graph, $E=\{e_i\}$ are directed edges, and $A=\{a_i\}$ are undirected edges. Our graph is ordinary and, therefore, does not contain loops and multiple edges. Briefly, a directed edge is depicted with an arrow from one node to another, while undirected edge is depicted with a simple junction between the two nodes. The junction can be otherwise depicted using two arrows: one arrow leads from node *u* to node *v* and another leads from node *v* to node *u*. (Fig. 1). Once, two nodes *u* and *v* in our combined graph are connected with undirected edge, these two cannot be connected with directed edge. (see Fig. 1, lower panel).

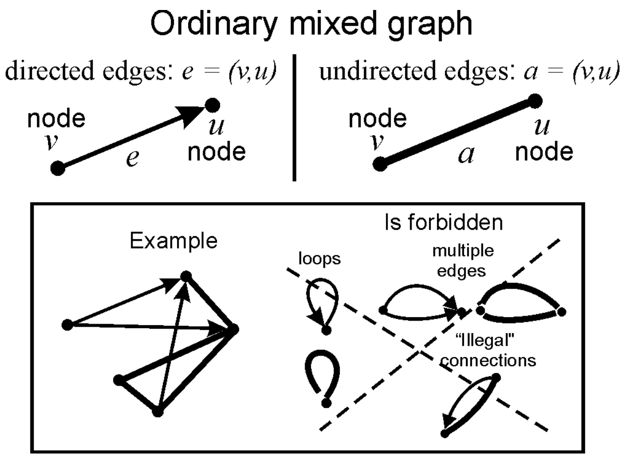

Figure 1.

Definition 1.

The graph is called Super Weak Computable when there is a closed route through all the nodes that uses both the directed and undirected (according the orientation) edges

$$v_0 \xrightarrow{a_0 \vee e_0} v_1 \xrightarrow{a_1 \vee e_1} \ldots \xrightarrow{a_k \vee e_k} v_0$$

Definition 2.

The graph is called Weak Computable when there is a route

$$v_0 \xrightarrow{a_0} v_1 \xrightarrow{a_1} \ldots \xrightarrow{a_k} v_k$$

following which one can pass through all the nodes using *only undirected* edges. So, it is clear: when the graph is Weak Computable, it is also Super Weak computable (see Fig. 2).

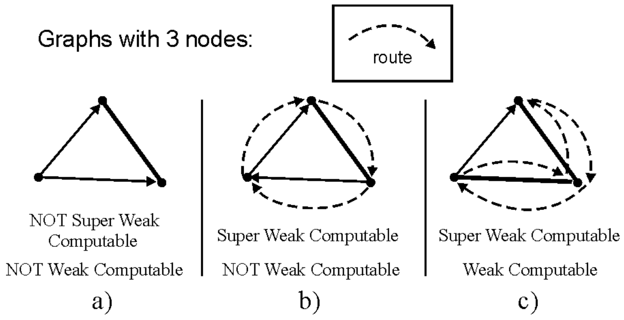

Figure 2.

Let the nodes of the graph be of three colors (A, B, and C).

$$v_i \in \{A,B,C\}$$

Further, we introduce two definitions.

Definition one: Transliteration of graph G $(\tilde{G})$ is defined as the change in a color map of graph G where all the nodes of graph G colored in C are recolored into color B, and all nodes colored in B are recolored into color C.

$$G(B \Leftrightarrow C) \equiv \tilde{G}$$

Definition two: Complement of graph G $(\bar{G})$ is defined as the change in a color map of graph G where all nodes colored in A are recolored into color B, and all nodes colored in B are recolored into A. Usually, this **complement** transformation rule is applied to the cases where there are no nodes colored in C.

$$G(A \Leftrightarrow B) \equiv \bar{G}$$

Let us enter the rule of transformation of colors in graph G:

$$G_t \rightarrow G_{t+1}$$

This rule depends on the existence of *at least one node colored in C* in the end of directed or undirected edges all of which start from the same node v_i. (Let us refer this condition as P-condition).

So, if P=0 (i.e., there are no nodes colored in C), then one can make following recoloring

$$P=0 \quad (I) \quad \Rightarrow \quad \begin{aligned} v_t = A &\Rightarrow v_{t+1} = A \\ v_t = B &\Rightarrow v_{t+1} = C \\ v_t = C &\Rightarrow v_{t+1} = B \end{aligned}$$

In the text below, we refer to this recoloring as rule I.

If P=1 (opposite situation), we make recoloring using following rule

$$P = 1 \;(II) \Rightarrow \begin{array}{l} v_t = A \Rightarrow v_{t+1} = C \\ v_t = B \Rightarrow v_{t+1} = A \\ v_t = C \Rightarrow v_{t+1} = B \end{array}$$

We refer to this recoloring as rule II.

Theorem 1. (About reversibility**).**

$$\tilde{G}_t \to \tilde{G}_{t-1}$$

Proof.

To prove the above theorem, it is enough to consider all five events (shown by numbers in circles) from Fig. 3.

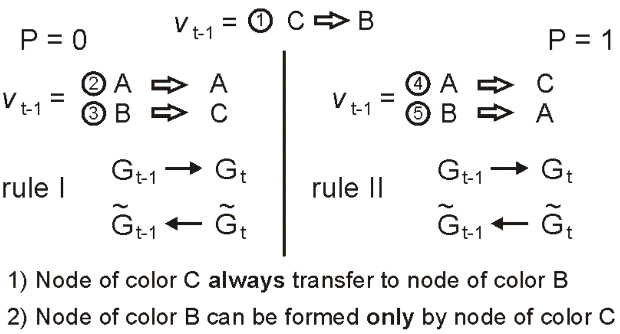

Figure 3.

Consider case (1). This case is obvious. If we do transliteration, then CB will be BC.

Consider case (2). Recoloring rule is rule I, so there are no nodes colored in C among the nodes of the graph where directed (or undirected) edges lead from node v_{t-1}. Therefore, in the derivative of this graph (i.e., in the graph that emerges after recoloring) there are no nodes colored in B. (Note here that the nodes colored in B can be formed only from the nodes colored in C). Consequently, in the transliterated graph, it will be no C nodes among all its nodes. This means that the transfer rule is still rule I. Also node v_t in graph $\tilde{G}_t$ will be colored in A in accordance with rule I. The rest three cases can be considered in the same way. The theorem is a simple consequence of two statements shown in Fig. 3.

Assume that recoloring of graphs starts at time $t = 0$. Also assume that the nodes have two colors: A and B at the start. This means that during the next step (there are no nodes colored in C; so rule I is applied to the entire graph) the graph will be converted into transliteration of itself, consisting only of A and C colors. (All nodes colored in B will be transformed into the nodes colored in C, whereas all nodes colored in A will keep its color). So, the graph will go along reversed-time path (see Theorem 1). Note: for each step taking place at time $t = n^* = -n + 1$, the graph will be the transliteration of the same graph at time n (see Fig. 4).

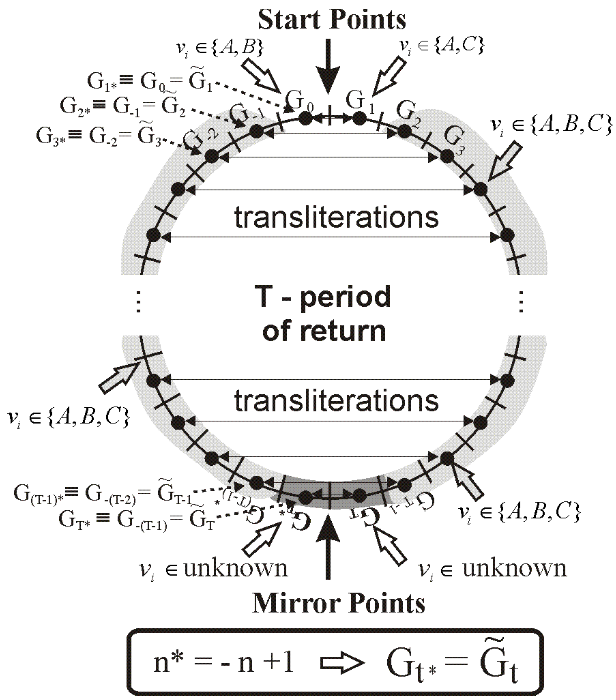

Figure 4. Propagation of recoloring of graph *G* with time.

Consider the time point located between the states of graph at time 1* and time 1 as the Start Point (SP) and continue the motion in two directions: let us called it "forward" time and "reversed" time. Due to the fact that our graph is defined as finite, these two motions (that is, changes in color of our graph with time) will meet at some time point T^* - T. The graph color "state" will be G_{T^*} at this point, where G_{T^*} is the transliteration of G_T. Let us refer to this point (graph state) as the Mirror Point (MP), and name T as the *period* (or the period of return). Here, we use the main property of *reversible finite Automata*: every such a construction "is obligated to pass through its starting point". That property is quite natural. If any recolored graph has only one predecessor (and it is easy to find this predecessor: let us make transliteration, then one step forward, then make transliteration again), then any recoloring of the graph is cyclic. Also, there is no "branch" that merges with this "cycle".

If $T > 2$, then the Mirror Point exists. In this case, full period of return ($2T$) is an even number. Otherwise, the graph in the Mirror Point is transliteration of itself. This means that all nodes of graph are colored in A, and graph simply stays the same.

Let us prove following theorems.

Theorem 2. (About coloring the Mirror Point).

Let $T > 2$ and the graph is Super Weak Computable. Let SP be the state between graph G_{1^*} that has colors A and B and graph G_1 that has colors A and C.

Then MP is the state between the graph G_T that has also only *colors A and B* and graph G_{T^*} that has only *colors A and C.* Besides, SP is not equal to MP.

Proof.

Let us start with the last statement that SP is not equal to MP. The Mirror Point is the *first time* when transliteration of G_{T^*} at time T^* equals G_T at time T. If it is not the case, we would fold in half our Fig. 4 and repeat the graph construction. So, we get a contradiction.

If at time T graph G_T has at least one node colored in C, then the original node where the edge leading to the node colored in C starts must be also colored in C. (*Recoloring rule II cannot be applied in the Mirror Point. It means that nodes colored in A are transformed into nodes colored in C, and the nodes colored in B are transformed into nodes colored in A. Yet, none of the nodes is transliteration of each other*). So, applying recoloring rule I and moving through all the nodes along the route leading to "another side", we can make a conclusion that all the nodes are colored in C. Then, the graph state changes from "*all nodes are C*" to "*all nodes are B*", and from "*all nodes are B*" to "*all nodes are C*" (T<3). Here, we immediately see the contradiction. So, theorem 2 is proven.

From this theorem, follows the theorem of *existing of number λ.*

Theorem 3.

If our graph is Weak Computable, then it follows that any v_i of its nodes when moving from SP to MP

$$v_{i,t=1} \rightarrow v_{i,t=2} \rightarrow \ldots \rightarrow v_{i,t=T}$$

is transformed into equal number of nodes colored in A (N_A), B (N_B), or C (N_C). Moreover, the number of nodes colored in B is equal to the number of nodes colored in C.

$$N_B = N_C \equiv N_{BC}$$

Let us introduce parameter $\lambda = N_A - N_{BC}$.

Proof.

Let us start from arbitrary node and move around all the nodes in the graph following undirected edges, and then return to the initial node. Every new step means "plus one" in our new numeration. It does not matter that some nodes may be passed several times on our way (see Fig. 5).

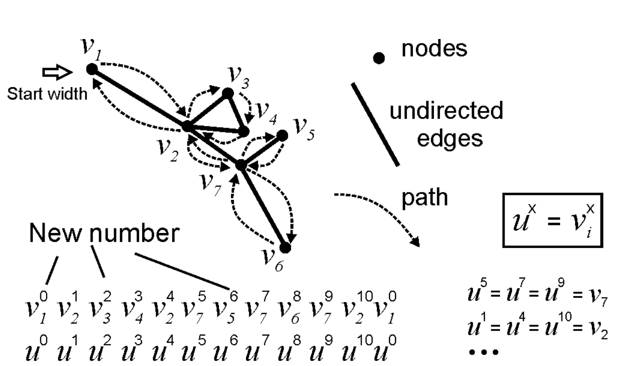

Figure 5. Our "numeration" of the nodes.

Let us arrange the nodes along a single horizontal line (circle). The most important thing in our numeration is that to the left and right sides of any node there will be a node that has a "tie" with another node through an undirected edge.

Let us look at Figure 6 starting from panel (a)

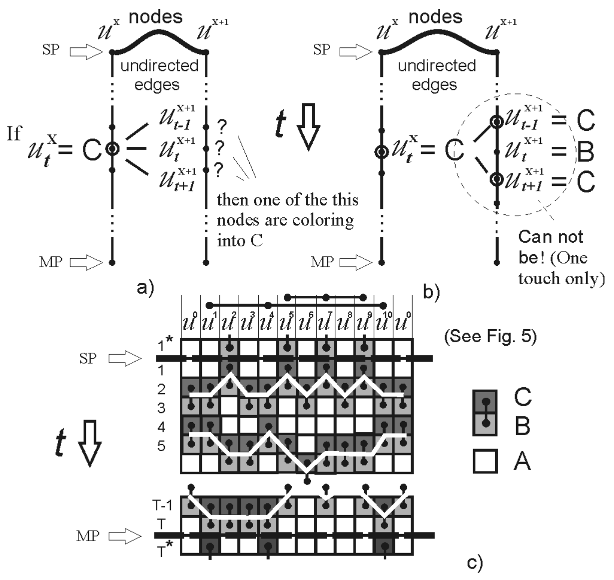

Figure 6. Illustration of Theorem 3.

Due to existence of the "directed half" edges leading from node u^{x+1}_t to node u^x_t and due to the fact that u^x_t is colored in C, then one of the nodes u^{x+1}_{t-1}, u^{x+1}_t, u^{x+1}_{t+1} will be also colored in C. Let us pretend that it is not the case and ask which color will be for the node marked by a central question mark (node u^{x+1}_t) in Figure 6a? If this node is colored in A, then the node beneath it (u^{x+1}_{t+1}) is colored in C. (Note that transfer rule here is rule II). This node cannot be colored in color B, either. Then node u^{x+1}_{t-1} above it is colored in C. Yet, node u^{x+1}_t cannot be colored in C. So, we have a contradiction here.

Then, we also note that a "double touch" is also not possible: u^{x+1}_t = B cannot be transformed into u^{x+1}_{t+1}= C. We must remind that here we use transfer rule II. So in this case, u^{x+1}_{t+1} must be colored in A. Again, we get a contradiction.

Therefore, the "double-touch" on either right or left sides is impossible! So, when moving from SP to MP, our figure consists of non-intersecting CB bands (see Fig. 6c). Thus, the theorem is proven.

Take Weak Computable graph. Consider graph G_{1^*}, consisting of colors A and B and having its own T and λ; also consider complementary graph $\overline{G}_{1^*}$ comprising only colors A and B with its own $\overline{T}$ and $\overline{\lambda}$. (We assume that $\overline{T} \geq T$, otherwise, just rename G_{1^*} and $\overline{G}_{1^*}$).

Let us take arbitrary node v for first graph G_{1*} and consider the entire array of colors $v(t)$ the node passes on its way from SP to MP. The size of this array is T.

Let us enter four arrays $a_v(t), b_v(t), c_v(t), \lambda_v(t)$; where $t = 0, 1, 2...T-1$.

Now, $a_v(t)=1$, if $v(t)$=A, and $a_v(t)=0$, otherwise. The same rules are true for $b_v(t), c_v(t)$.

So

$$\lambda_v(t) = \sum_{0\le p\le t}^{p} (2a_v(p) + b_v(p) + c_v(p) - 1)$$

Let us define three arrays $C_v(k), A_v(k), f_v(k)$, and let us call array f as "integral phase".

$C_v(k)=0$; but if we can find t which makes value $k=\lambda_v(t)/2$ an integer number and therefore $v(t)=C$, then $C_v(k)=1$.

If $C_v(k)=1$, then we can write that $f_v(k)=t$. Otherwise, we can assume that it is "undefined": $f_v(k)=-1$.

We can determine properties of array $A_v(k)$ in the same way as we did for $C_v(k)$ array: $A_v(k)=0$; but if we can find t which makes value $k=(\lambda_v(t)-1)/2$ an integer number and therefore $v(t)=A$, then $A_v(k)=1$. (By analogy with $A_v(k)$ we can define array $B_v(k)$, but it is easy to show that it coincides with $C_v(k)$).

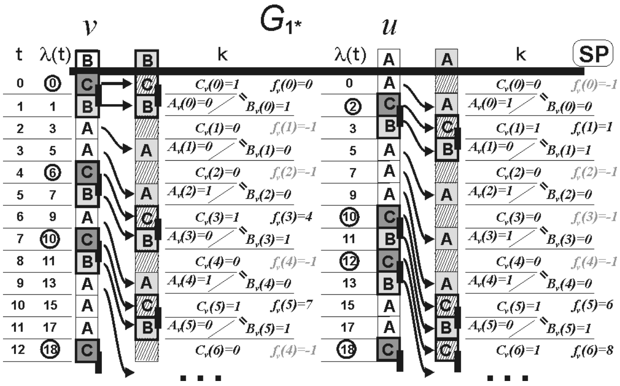

Fig. 7. Examples of constructing arrays $C_v(k)$, $A_v(k)$, $B_v(k)$, $f_v(k)$. For node $v(t)$ with the beginning: B (SP) CB AA CB A CB AAA C and for node $u(t)$ with the beginning: A (SP) A CB AAA CBCB AA C. From these bands, we can construct their secondary bands where the cells A are transferred "through" an empty cell. Because C and B are always in pairs, values for λ(t) for the cells C are always even (see circled λ(t) in this figure).

Now, we can define following arrays $\overline{G}_{1*}$ for coloring:

$$\overline{a}_v(t), \overline{b}_v(t), \overline{c}_v(t), \overline{\lambda}_v(t), \overline{A}_v(t), \overline{C}_v(t), \overline{f}_v(t).$$

Finally, we introduce an important definition.

Consider the case where value $T+\overline{T}$ can be divided by 3 with no residual ($K=(T+\overline{T})/3$) and assume that all eight conditions are fulfilled as shown below.

First three conditions are:

$$G_T = \overline{G}_{\overline{T}} \qquad [1]$$

$$\lambda = -\overline{\lambda} \qquad [2]$$

$$\overline{T} - T = \lambda \qquad [3]$$

Next four conditions are:

$$C_v(k) + \overline{C}_v(k) = 1 \qquad [4]$$

$$A_v(k) + \overline{A}_v(k) = 1 \qquad [5]$$

$$\overline{A}_v(k) = C_v(k) \qquad [6]$$

$$\overline{C}_v(k) = A_v(k) \qquad [7]$$

for any node v and $k = 0, 1, ..., K-1$.

Additionally, from ratio [4] we can define full integral phase with modulus 2

$$F_v^{(2)}(k) = \begin{cases} f_v(k) \bmod 2 & \text{if } f_v(k) \neq -1 \\ 2 + \overline{f}_v(k) \bmod 2 & \text{otherwise} \end{cases}$$

So we can write the last (eighth) condition as:

$$\begin{aligned} &F_v^{(2)}(0) \equiv 0 \pmod 2 \\ &F_v^{(2)}(2k-1) \equiv F_v^{(2)}(2k) \pmod 2 \qquad [8] \\ &\textit{and if K is even} \\ &F_v^{(2)}(K-1) \equiv F_u^{(2)}(K-1) \pmod 2 \end{aligned}$$

for all nodes v and u and for any $0<k<(K+1)/2$.

If all the above conditions are met, then we can make a conclusion that graphs G_{1*} and $\overline{G}_{1*}$ move from SP to MP with conservation of the *Invariant of Precise Filling (IPF).*

3. "X-problem of number 3" in one dimension. Definition

Consider that L nodes of the graph are placed on a circle and numbered as $x = 0, 1, \ldots, L-1$.

The edges of the graph are defined by two integer numbers: n (conventionally "left") and m (conventionally "right").

There exist only edges $(x,(x-(i+1)*b_i(n)) \bmod L)$ and $(x,(x+(i+1)*b_i(m)) \bmod L)$ for all x and i when function b_i is significant. Here, $b_i(n)$ is the i-th bit of binary expansion of number n: $n=b_0(n)+ b_1(n)*2^0 + b_2(n)*2^1 \ldots + b_p(n)*2^p$). "*Mod L*" shows that the circle is continuous and closed (no gaps). Let us refer to such a construction as *Automat of Configuration 2x3* (AC23) in one dimension.

Suppose that the above introduced integers n and m are odd. Then, our Automat is Weak Computable. It is so because it contains all undirected edges (x, $(x+1)$ *mod* L). One can also see that if the mask contains bits $b_p(n)$, $b_{p+1}(n)$, $b_p(m)$, $b_{p+1}(m)$ = 1 (e.g., masks (6,7), (6,14), (6,22)), then this mask (Automat) is Weak Computable too.

Suppose that there exist such n an m that Automat at any L having any starting colors G_{1*} and $\overline{G}_{1*}$ always goes from SP to MP while keeping IPF constant. Let us call it a "correct mask". In other cases, the mask is "incorrect".

From this point, we can constitute the "X-problem of number 3" in one dimension.

We can prove (or disprove) that the masks (1,1), (1,3), (3,5), (3,3), (5,5), *etc* are correct. (For example, we know that mask (1,5) is incorrect). A full list (more precisely, apparently full list) of correct masks till the values n, m = 39 is shown in Figure 8.

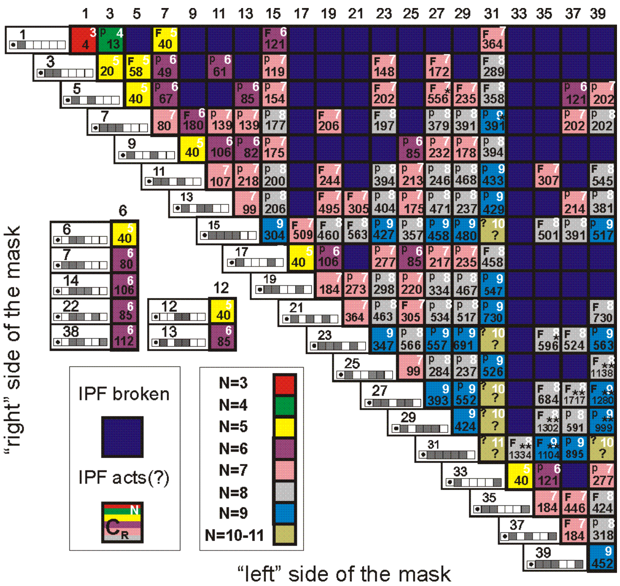

Figure 8. The matrix showing which of Weak Computable masks is correct. Correct masks are painted in different colors depending on N, where N is the total number of pixels in the mask plus one. (Note that we include a "central" point into our mask). Incorrect masks are shown in dark blue. All the central-symmetry cells (the cells on the main diagonal of the matrix) are correct. In cells with N <10 denotes the value c_R (with small c) for each masks. Since all the values C_R are odd we have introduced the designation c_R = (C_R-1) / 2. An asterisk (*) indicates that the test has not been completed. ("End of test" – that is determined entirely all 6 sub-tables, and found that they pass each other by substituting from a figure 11). However defined three sub-tables. Two stars – defined less sub-tables (See explanation in the text).

First, we illustrate the fulfillment of the eighth condition (see descrption of the conditions in the text above) for the correct masks (see Fig. 9).

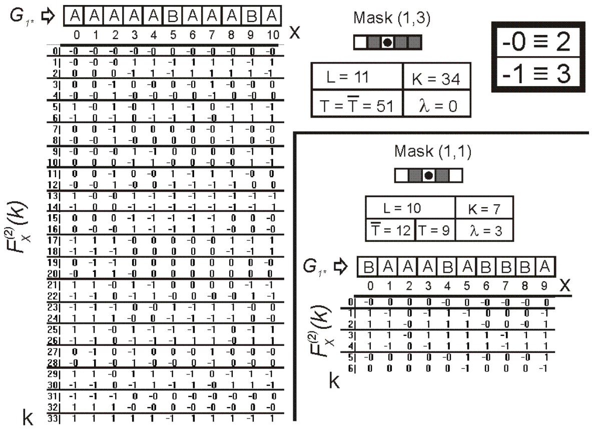

Figure 9. Illustration of fulfillment of the eighth condition for correct masks (1,3) and (1,1).

It is obvious that after the first row, the filling of the table is going by "pairs of lines". In Fig. 9, the last line number (k=33) for mask (1,3) is odd. This number should not necessary be odd, it can be also even.

Let us draw our attention to outgoing and very powerful observation, which, perhaps, will help us to solve the "X-problem" some day.

Let us consider a complete integrated phase with modulus 3. So, let function $F^{(3)}_x(k)$ be defined as

$$F^{(3)}_x(k) = \begin{cases} f_x(k) \bmod 3 & \text{if } f_x(k) \neq -1 \\ 3 + \overline{f}_x(k) \bmod 3 & \text{otherwise} \end{cases}$$

Then we note that the numbers in any line (except in the first one) are unambiguously determined from the previous line, facing a number $F^{(3)}_x(k)$ of points of the mask directly above it. Here, the points of the mask also include a "central" point. The number of all configurations leading to values 0, 1, 2, 3, 4, 5 *are equal* (see Fig. 10). We refer to this algorithm as *Resolution*; then we refer to the corresponding table for numbers 0, 1, 2, 3, 4, 5 as *Resolution Table* (RT); finally, we refer to number C_R of rows in each sub-tables as *Resolution Constant* for each "correct mask".

So, having the first line and the Resolution Table, we can straightforwardly and without difficulty restore the entire function $F^{(3)}_x(k)$. Therefore, we can restore the matrices C_v and A_v too. The criterion that we have reached the end (that is, we have reached MP), is precisely that $F^{(3)}_x(k) \bmod 3 = 0$ along the entire line (i.e., for all x).

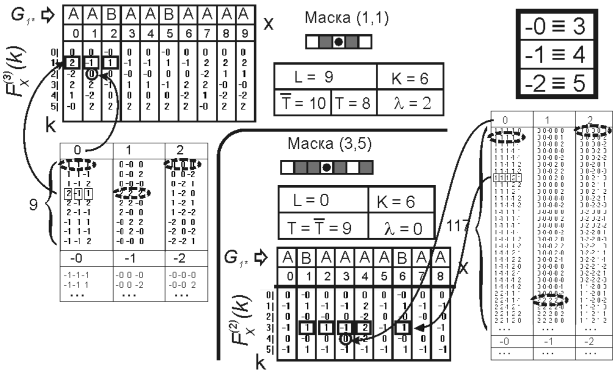

Figure 10. Illustrations to main observations made for masks (1,1) and (3,5). This figure shows the Resolution Table for mask (1,1) and initial lines of the Resolution Table for mask (3,5). We show the Resolution Table for numbers 0, 1, and 2 only. The Resolution Tables for numbers -0 (3), -1 (4), and -2 (5) can be obtained automatically, as "complementary" to the original RT.

4. Resolution Table for correct masks in one dimension. The first observations

4.1. First and foremost observation.

It is clear that the Resolution Table has very simple symmetry. In the case that we have only one of six (=0, =1, =2, =3(-0), =4(-1), =5(-2)) sub-tables, we can immediately determine five remaining sub-tables. To do this, we make a substitution as shown in Fig. 11. (In Fig. 11, the "base number" for each of the sub-tables is shown in gray. Each sub-table contains the base number in all the cells. Such variants are shown by a dashed-line oval in Fig 10).

=0	=1	=2	$=\bar{0}$	$=\bar{1}$	$=\bar{2}$
1	2	0	$\bar{1}$	$\bar{2}$	$\bar{0}$
$\bar{1}$	$\bar{0}$	$\bar{2}$	1	0	2
2	0	1	$\bar{2}$	$\bar{0}$	$\bar{1}$
$\bar{2}$	$\bar{1}$	$\bar{0}$	2	1	0
0	1	2	$\bar{0}$	$\bar{1}$	$\bar{2}$
$\bar{0}$	$\bar{2}$	$\bar{1}$	0	2	1

Figure 11.

Let us take a sub-table to output the numbers (=0), and we just call it a Resolution Table (*RT*).

So, RT is the set of C_R rows $\{a_0, a_1, \ldots, a_{N-1}\}$, where $a_i \in \{0,1,2,3,4,5\}$ (For numbers 3, 4, 5 we will use the old notation, or $\bar{0}$, $\bar{1}$, $\bar{2}$ or -0, -1, -2 where it is comfortable). Note that we have an "isolated position" in our mask: it is a central point.

Let us put *column number 0* as the central point in the Resolution Table. In order to establish a correspondence between the points of the mask and the columns just list the points of the mask from left to right. For instance, considering mask (3,5) we obtain: "column 0" in the table – the central point (offset 0), then for "column 1" – the point with offset -2; for "column 2" – the point with offset -1. Further, for the "3 column" (here we should jump over 0) – the point with offset 1. Finally for the last column, we have the point with offset 3.

There is one more symmetry. If the mask is mirrored with respect to the central point, the same correspondence is kept between the column and the points of the mask: that is, the column which offset was x is now assigned to a point with offset $-x$ and so on. Eventually, we will end up with the same Table. Most likely, this symmetry is trivial.

4.2. About the contents of RT.

The masks are divided into three classes in accordance with the numbers in their Resolution Tables.

4.2.1. *Small* class masks (RT contains only the numbers 1, 2, 4(-1)). This class is depicted by the lack of any symbol in the top-left corner in Fig. 8.

4.2.2. *Middle* class masks (RT contains the numbers 1, 2, 3(-0), 4(-1), 5(-2). This class is depicted by letter ”*p*” (*p* stands for partial) in the top-left corner in Fig. 8.

4.2.3. *Full* class masks (RT contains all numbers 0, 1, 2, 3(-0), 4(-1), 5(-2). This class is depicted by letter “*F*” in the top-left corner in Fig. 8.

Small-class masks include (apparently?) all masks with central symmetry (it was checked for $n < 40$).

We denote the sum in RT in zero column of number 0, 1, 2, 3, 4, 5 as s_0^{+0}, s_0^{+1}, s_0^{+2}, s_0^{-0}, s_0^{-1}, s_0^{-2}. We denote the sum of number 0, 1, 2, 3, 4, 5 in other columns as s^{+0}, s^{+1}, s^{+2}, s^{-0}, s^{-1}, s^{-2}.

These values for masks (n, m) with odd n, m <19 are shown in Figure 12.

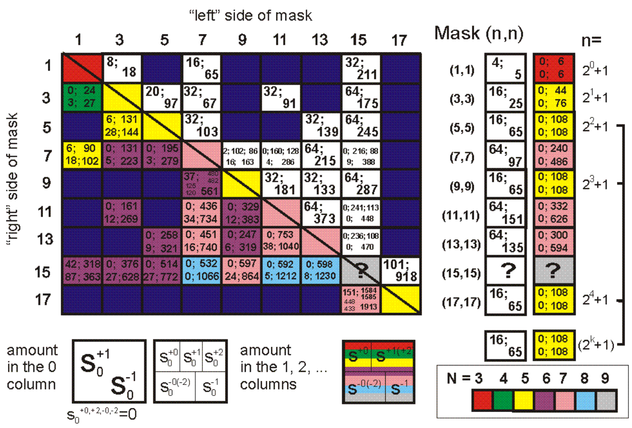

Fig. 12. Summary of the numbers 0, 1, 2, 3(-0), 4(-1), 5(-2) in RT. (N-total is number of points in the mask, including the central point).

We can say that the correct masks are divided into *completely* correct (that is, those for which $s_0^{+0,+2,-0,-2} = 0$) and *not completely* correct.

Moreover, we divide completely correct masks into completely correct of the first kind (masks with $s_0^{+1} = 2^{N-1}$ where the sub-table with zero column is equal to a simple enumeration of all possible combinations of +1 and -1), and of the second kind (otherwise).

Completely correct masks of the first kind are all symmetric masks for n <15, and masks (1,3), (1,7), (3,7), (5,7), and (3,11).

Completely correct masks of the second kind are masks (3,5), (15,17).

We denote RT of the mask (n, m) as [n, m], simply by using parenthesis instead of brackets.

We also point out that there is the identity between following RTs: [5,5] = [9,9] = [17,17] = ... = $[2^k+1, 2^k+1]$ where $k > 1$.

4.3. Building Resolution Tables for masks of type $(1,2^k-1)$.

First, we should number the columns. Let us do it in accordance with the rules described in paragraph 4.1 of this chapter (we put column number 0 as the central point, and so on).

Now let us sort out the rows. Assume that we have row (**-1**, 2, 0, -2, 1). After we transfer the “0-“ column (that is the first element, “-1”, of the given row) from the first position to the last position, we obtain following string: (2, 0, -2, 1, **-1**). Now consider this string as a record number in a *heximal* number system, where the bits are counted from right to left: (2, -0, -2, 1, **-1**) = $4*6^0 + 1*6^1 + 5*6^2 + 3*6^3 + 2*6^4$ = 4 +6 +180 +6048 +2592 = 8830. Finally, we reshuffle the rows in accordance with the ascending order of these numbers.

At his point, Resolution Table for masks $(1, 2^k-1)$ can be easily constructed by the method of induction based on k (see Fig. 13 for details)

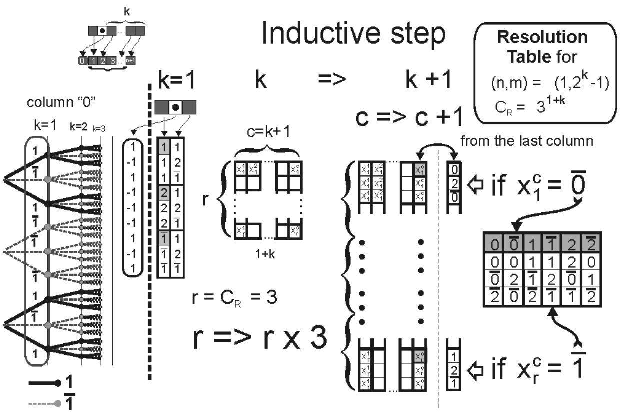

Figure 13. Building a Resolution Tables for mask $(1, 2^k-1)$ by method of induction.

A zero column is obtained by simple linear fractal of the numbers 1 and -1 (see Fig. 13, zero column is shown on the left). Figure 13 shows the Resolution Table (see table on the right) for k = 1. Now, we construct the induction step for other columns. In one step, the number of rows increases by a factor of 3 (C_R increases three times), but the number of columns increases by a factor of one.

Let us follow operations as shown in Figure 13. The table can be "tripled" using a trivial method, and the new column is obtained from the previous one in accordance with the table below (see Fig. 14). It can be shown that the obtained mask is the completely correct mask of the first kind.

Also it can be emphasized that we obtained *all* Resolution Tables for *solid* masks, that is masks $(2^k-1, 2^p-1)$ where k, p > 0, and RT masks $(1, 2^{k+p}-1)$ *include* the entire series with constant k+p. Yet, more will be shown in the next paragraph.

4.4. Compatibility of Resolution Tables for various correct masks with the same number of columns.

Let us consider several masks with the same N (we remind here that N is the total number of pixels in the mask, including the central one), and check each rows of the Resolution Tables for these masks to answer the question: whether or not its converted row appear in some other sub-tables which we already have calculated. If the answer is yes, then masks are not compatible.

Our observation is as follows: if we consider that the columns are numbered in accordance with “symmetry consideration” stated in 4.3, then all the correct masks with the same N *will remain compatible*. That is, for each N there is its own "integral" Table $R^{(N)}$, with a corresponding number of lines in it. Observation has been tested for N = 4, 5, 6.

Let us begin with N = 4 (see Fig. 14). For N=4, we have only two correct masks (1,3) and (3,1) Their C_R=27. C_R=37 is their association (Resolution Tables intersects at 17 rows). Remember, removing (rearranging) the central point is strictly necessary! Otherwise, even in the case where N = 4 there will be numerous violations!

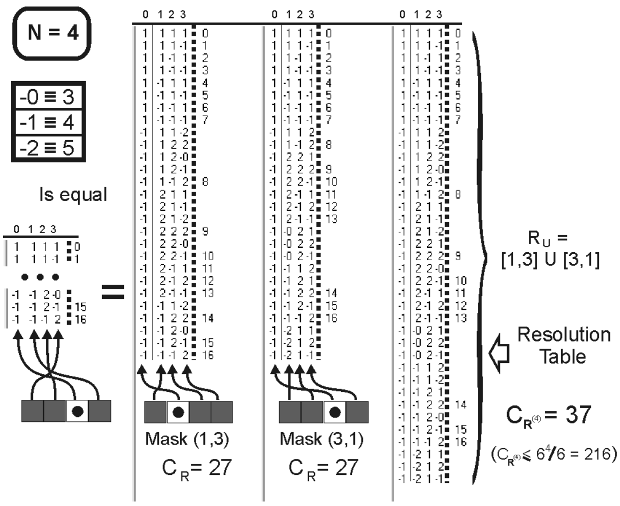

Figure 14. The construction of the integrated RT for N = 4.

Note again that if we take the most left column numbering in Fig. 14, according the "trivial" symmetry in paragraph 1 of 4.1 (the column with the shift x for mask (1,3) corresponds to a column with a shift $-x$ for mask (3,1)), then we would come to RT identical to the first table. Our association is not trivial.

Consider the case $N = 5$ (see Fig. 15). Apparently, almost all correct and "obvious" Weak Computable masks are shown in this Figure.

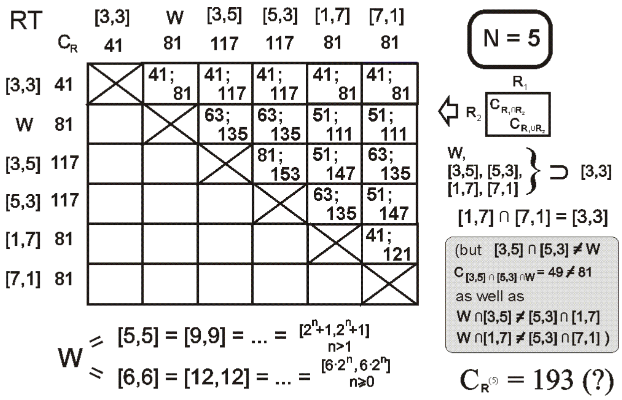

Figure 15. The construction of the integrated RT for N = 5. The cells in this table contain two numbers: in the upper left corner – C_R for the intersection of RT of the masks, and at the bottom right corner – C_R to combine.

Figure 15 shows two infinite series of masks with the same RT: masks from the first series have the same RT: W = [5,5] = [9,9] = [17,17] ... = [6,6] = [12,12] = [24, 24]… Also we have 5 correct masks [3,3], [3,5], [5,3], [1,7], [7,1]. All other possibilities such as masks (3,9), (3,17), (5,9), and (5,17) are incorrect.

The case with $N = 6$ is much more diverse (Fig. 16).

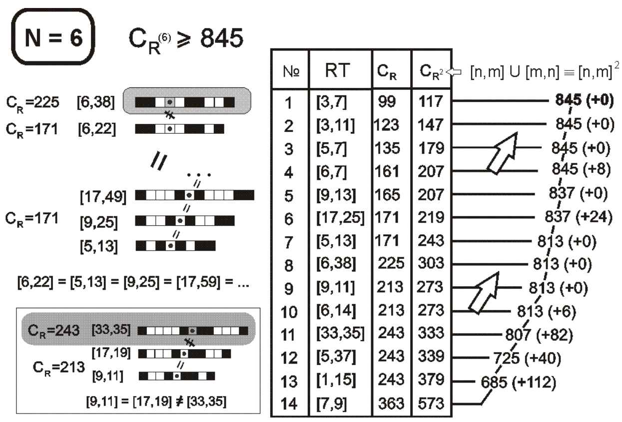

Figure 16. Left side – three identities for masks with N = 6. Right side – the steps necessary to calculate integral C_R.

Figure 16 reveals a (seemingly endless) line of masks having the same RT: [5,13] = [9,25] = [17,49]. Stand-alone mask (6,22) has the same RT as the masks along this line. Yet, contrary to the case with $N = 5$, there is no second line (another series). Note also that a "stand-alone" identity [9,11] = [17,19] does not have a continuation.

The right side of Fig. 16 shows the steps necessary to implement to provide integrated table $R^{(6)}$. We used the correct mask with $N = 6$, odd n, m <40, then we added three correct masks with even n ([6,7], [6,14], [6,38]). After that, we sorted the masks out in accordance with ascending C_R. Then, every RT of mask [n, m] was combined with centrally symmetric RT of mask [m, n]. Eventually, we started to fold the masks, starting with the mask having the largest C_R. The results are shown in Fig. 16. The final number (845) cannot be much lower than the accurate one.

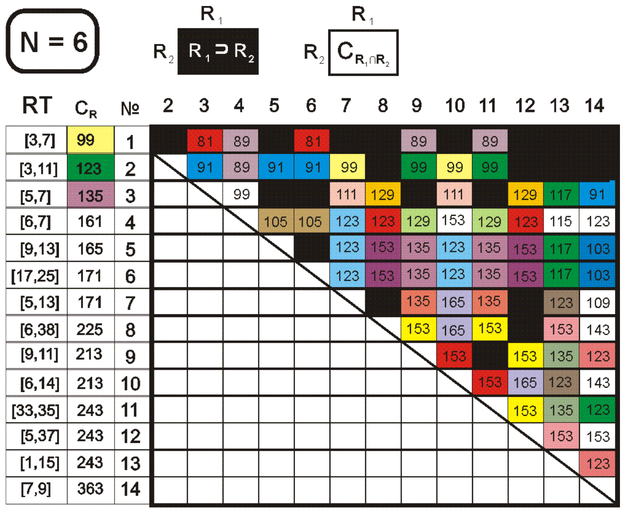

Figure 17. "Table of Coincidences" for Resolution Table for masks with N = 6.

Fig. 17 shows the "Table of Coincidences" for the intersection of RT of our masks. Black color of the cell means that one mask includes another (the upper mask includes the left one). Otherwise, cells show C_R (number of rows) in the intersection table. If these values are the same and the cells are painted in the same color (but not in white!), it means that there is a coincidence among not only the number of the rows in those masks, but also among the RT numbers. Examples are as follows: [3,11] = [33,35] ∩ [7,9]; [6,38] ∩ [9,11] = [33,35] ∩ [5,37]; [6,7] ∩ [9,13] = [17,25] ∩ [6,14], and so on.

It can be seen that the algebra of Resolution Tables is very intricate.

5. Conclusion

There is no doubt that similar "*X-problem of number 3*" exists *in all dimensions* [7].

Using a personal computer (PC), we tested the following amazing statement: each and every mask in one and two dimensions, which includes all the points adjacent to the "central" point (we refer to such mask as simple) and demonstrates the property of central symmetry (that is, is symmetric with respect to the central point) is correct.

After several months of PC operation, we did not find a single exception! (We used the "light" test, that is, we checked only the first three of the nine statements. We found that building *F* arrays in two dimensions has been rather challenging. So, it is impossible to guarantee that this statement is true. Some other interesting aspects of the *"X-problem of number 3"* in two dimensions can be found in paper [8].

Readers can download illustrative program from kornju.hop.ru.

6. Acknowledgements

We thank Dr. Eugene Moskovets for reading and correcting this manuscript.

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